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INVESTIGATION OF THE THERMAL MODE OF A HIGH-SPEED ELECTRICAL CONTACT BY MATHEMATICAL MODELING METHODS

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The temperature mode of a metal contact upon transition to a hybrid and further on to a plasma one has been studied. Spatial (three-dimensional) mathematical modeling confirmed the existence of a dependable metal contact before melting of the anchor material and transition to an arc mode of passage of the current after the onset of boiling. The possibility of destruction of the accelerated body during the acceleration time has been investigated.

Let us consider the problem of mathematical modeling of acceleration of conducting macrobodies in pulse electrodynamic accelerators of the railtron type [1, 2]. The simplest electrical diagram and the typical cross section of the channel of such an accelerator are shown in Fig. 1. The railtron guides (rails) carry the electrical current, which closes the circuit of the current source through a movable conducting jumper anchor. The magnetic field set up by the current of the rails interacts with the current in the anchor and generates the Lorentz force, which pushes the anchor along the rails. As a result, the anchor is accelerated.

The basic advantage of the railtron over the commonly used powder or light-gas accelerators lies in the absence of fundamental limitations on the hurling velocity. The range of reproducible results obtained with acceleration of solids in the railtron reaches 8.5 km/sec. Still, when the metal anchor is accelerated by the Lorentz force, reproducibility of the results is limited by velocities of 2.5–3 km/sec. In this case, attainment of a velocity on the order of 1 km/sec entails a disturbance of the metallic conduction in the sliding contact between the rail and the anchor [3, 4]. This leads to erosion of the contact and to limitation of the service life of the accelerating channel. Study of the dynamics of the contact breaking would allow the development of appropriate measures for increasing the velocity of projectiles in such devices of the simplest structure.

Experimental and theoretical investigations carried out at various research centers [3, 4] showed that, at the beginning of acceleration, when the velocity is yet below several hundred meters per second, there is a stable contact between the anchor and the rails. At velocities of 500–1000 m/sec, an unsteady mode of passage of the current through the contact surface exists practically over the entire acceleration length. An arc discharge is originated. The range of velocities at which this occurs depends within some limits on the current, initial velocity, materials of the electrode and anchor, its configuration, etc. The point of transition to an arc discharge was identified by the researchers as a "crisis" of the metal contact.

According to [3–5], erosion begins from angular points to the aft edge of the anchor and propagates in the direction of its movement in the form of a crescent-shaped wave. Simultaneously, the current distribution shifts forward to the zone of the metal contact. When the metal anchor was accelerated to a velocity over 2 km/sec, the arc stage occurred within 350 μsec of the start [3, 4]. Also, the anchor destruction during acceleration was noted. This was identified from the appearance of a few cavities on the target, which corresponded to individual parts of the anchor.

Because of numerous difficulties in the experimental study of processes in a high-speed electrical contact, the erosion mechanisms can be analyzed only with the aid of mathematical modeling taking into account the contribution

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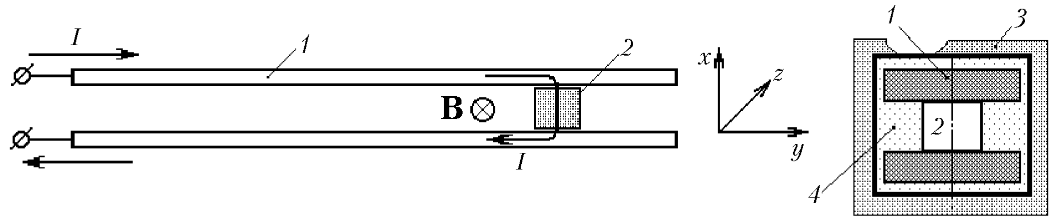

Fig. 1. Basic diagram of the railtron: 1) guide and conductor rails; 2) accelerated body (an anchor or another current armature); 3) power bandage of the channel; and 4) insulator.

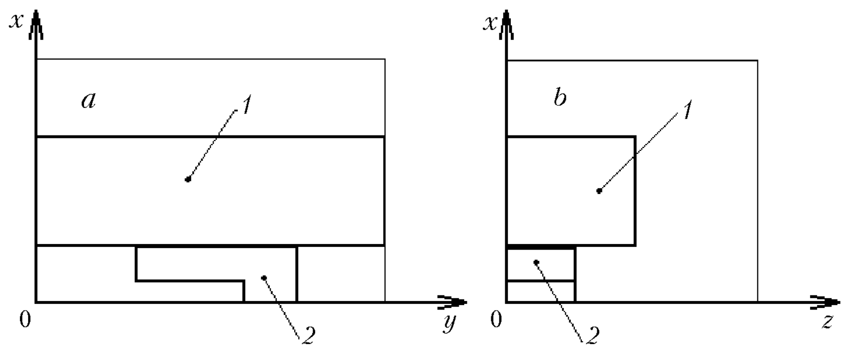

Fig. 2. Schematic of the railtron section by planes z = const (a) and y = const (b): 1) rail, 2) U-shaped anchor.

of various processes (the current distribution, the material heating, and phase changes) in a substantially multidimensional moving region. In order to study the dynamics of these processes, mathematical modeling has been made of the anchor acceleration and heat mode of the sliding current surfaces. The results of mathematical modeling have been compared with the experimental data. The formulation of the problem is on the whole determined by the need for investigating accelerators of the railtron type with a complex topography of the anchor and channel.

Mathematical Model and the Method of Numerical Modeling. Electromagnetic fields were described using the so-called quasi-stationary, or MHD, approximation [6] the Maxwell equations. A closed spatially three-dimensional time-dependent model [6, 7] combines the calculation of the current distribution in conductors and the local heat release with a direct computation of the accelerating force. The model is uniform over various subregions with markedly different types of electrical conduction (a conductor or a dielectric). The diagram of the model spatial region used in calculations is presented in Fig. 2. The anchor moves here in the positive direction of the y axis.

When bodies are accelerated in the railtron, the most complex and interesting phenomena occur in the vicinity of the anchor whose characteristic length is comparable with the cross section of the channel. Therefore, in modeling it is expedient to limit the considered space and describe the fields in the region rigidly connected with the sliding anchor. The length of the region (in the direction of the y axis) amounts to several calibers of the accelerator on both sides of the anchor (Fig. 2). Therefore, in the calculation we will consider not the entire three-dimensional accelerator but rather the part of it falling at the region rigidly connected with the anchor and moving with it. Symmetry of the calculation region makes it possible to construct the solution based on only a quarter. The development of the model draws on the marked difference in the length of the accelerator (along the y axis) and its transverse dimensions. It is also taken into account that the only electromagnetic quantity specified externally can be taken to be the total current determined by the power supply. With this approach, the problem arises of assigning boundary conditions on the front and rear boundaries of the considered region. On its lateral boundaries this problem is absent, since the railtron channel is generally enclosed in a conducting power bandage. Therefore, it is natural to examine a model where at the ends of the calculation region tangential components of the magnetic field are specified, which correspond to an infinitely long (along the axis y) system of semiconductors; the total current is assigned for each of them. This field is the solution of a corresponding spatially two-dimensional problem. As a result, to assume the boundary conditions it is necessary to solve two special problems for integro-differential equations at the ends of the calculation region [7]. The three-dimensional problem is solved using the algorithm [6, 7] with respect to specified tangential components of the magnetic field.

In the current study, algorithms of assigning the boundary conditions generalized for various structures of the accelerating channel have been constructed. Furthermore, novel algorithms and programs have been developed, which allow obtaining numerical results corresponding to a specific engineering object. They make it possible, in particular, to investigate degradation of the metallic conduction. In our case, calculated results and experimental data are in good qualitative and quantitative agreement. It is assumed in the problem that the electric conductivity of materials is temperature-dependent. In the conducting part, phase changes are possible, namely, melting and boiling.

Details of the mathematical model and computational algorithm are given in [6, 7]. Below, use is made of the notation traditional for works on modeling of electromagnetic phenomena. It is the same as that adopted in [6, 7]. In modeling, the emerging system of linear algebraic equations is solved using the method of conjugate gradients combined with an incomplete Cholessky factorization [8–10].

The calculation region is described using a set of logic arrays completely determining the region, its boundary, and thus the matrix of the system of linear algebraic equations (i.e., the difference scheme) of the problem solved [7, 11]. It is fairly difficult to specify the values of arrays on the region boundaries. Within the framework of this study we have developed and implemented, based on computer programs, a novel (universal for various configurations of the object of investigation) system of forming logic arrays at the edges of boundary cells. The result of employing such arrays is the problem for a discrete vector potential (in terms of which electric and magnetic field vectors are expressed, as well as other quantities referred to the grid edges, faces, and cells [6]). The heat mode was modeled using the traditional approach [4–6] and the model providing a stable solution.

Numerical Modeling of Evaporation. In the model [4–6] in conducting regions use is made of temperature dependences of the electric conductivity, thermal conductivity, and specific heat with account for phase changes. This makes it possible to determine temporal boundaries of melting and boiling, and the instant of onset of evaporation of the conductor material. In the case of evaporation of the cell, its electric conductivity should sharply decrease and, ideally, go to zero. However, when the algorithm [6] is used in this case, the difference scheme should also be restructured as a result of the change of the boundary between conducting and nonconducting subregions. Previously, this version (with a change of the boundaries of conducting and dielectric subregions) was not used. In modeling the evaporation of a conductor the electric conductivity in the evaporated part was assumed [4–6] to be equal to a small "background" value (several orders of magnitude smaller than the value prior to evaporation). However, with this approach, conducting subregions with a low electric conductivity are formed, which should correspond to dielectric ones. The use of a conductor with a low electric conductivity for modeling a dielectric can strongly deteriorate the solution stability [6]. For example, in the case of a homogeneous conducting subregion with a low electric conductivity, the dependence of the solution on the right-hand side contains a low electric conductivity in the denominator [6]. If the magnitude of the "background" electric conductivity is taken to be large, the heat release and current distribution are calculated incorrectly. With a small value of the electric conductivity, the solution becomes strongly dependent on insignificant changes on the right-hand side of the system of equations [6, 7], which unavoidably leads to an unsatisfactory convergence of iterations and to a sharp decrease in the time step.

To avoid these difficulties, the following model has been proposed in the current study; when the boiling temperature in a conducting cell is exceeded, this cell is replaced with a dielectric one with an appropriate restructuring of logic arrays [7, 11] used for describing the calculation region and difference schemes.

We next consider in greater detail the question as to stability of the solution for various methods of the evaporation modeling. Consideration will be given to conditionality of the system of linear algebraic equations obtained with a difference approximation of Maxwell equations. The minimum and maximum eigenvalues will be calculated using somewhat modified versions of the methods [8, 12].

With no movement, the matrix of the system of linear algebraic equations of the difference scheme [6, 7] is symmetric. Here, the conditionality number is the quotient obtained when the maximum eigenvalue is divided by the minimum one. In the case of movement, the matrix is asymmetric and the conditionality number is the square root of the ratio between the maximum and minimum eigenvalues of an auxiliary matrix, which is equal to the product of the matrix of the system and its conjugate [12].

TABLE 1. Eigenvalues of the Matrix M for Various Methods of Modeling the Material Evaporation

Version number	μ_{max}	μ_{min}	Conditionality number
1	20.457	$5.331 \cdot 10^{-5}$	$3.837 \cdot 10^{5}$
2	20.457	$6.906 \cdot 10^{-9}$	$2.962 \cdot 10^{9}$
3	20.457	$5.326 \cdot 10^{-5}$	$3.841 \cdot 10^{5}$

Given below are the maximum and minimum eigenvalues of the matrix M of a system of linear algebraic equations obtained with the difference approximation of Maxwell equations on a three-dimensional difference grid for the considered problem with no movement.

We now calculate eigenvalues for various versions of modeling of the material boiling and evaporation: 1) there is no boiling in the region, 2) the evaporation temperature is reached in the two outermost cells of the difference scheme at the anchor edge (parameters of the problem vary according to the traditional scheme [4–6], and the "background" electric conductivity is introduced), and 3) the two outermost cells of the difference scheme at the anchor edge become dielectric (the material evaporates and the difference schemes are reconstructed by the algorithm proposed). In the construction of the matrix M for all versions in the approximation of the time derivative, the time step is the same. Iterations in the calculation of eigenvalues were continued until the discrepancy $\|\mathbf{MA} - \lambda\mathbf{A}\|$ became smaller than 10^{-10} at $\|\mathbf{A}\| = 1$.

From the tabulated results (Table 1) it is seen that the reconstruction of difference schemes in the material evaporation makes it possible to improve the conditionality of the system being solved by a few orders of magnitude. It should be noted that the conditionality number of version 2 is determined by the value of the "background" electric conductivity: the smaller it is, the larger this number, tending to infinity when the "background" quantity tends to zero. In calculations of this number for various versions of the "background" electric conductivity we have obtained corresponding results. The maximum eigenvalue is practically independent of the model version. For the case with movement, the conditionality of the system changes similarly, though the eigenvalues are dependent on the anchor velocity. Thus, in studying the processes in accelerators, a model with restructuring of the descriptors of boundaries and difference schemes in the material evaporation in cells is preferred.

Results of Mathematical Modeling. We now present some results of numerical modeling of acceleration of the metal anchor up to a velocity over 2 km/sec, which continue studies [4, 5]. Calculations of the distributions of various fields were performed for a U-shaped anchor. In this case, the length of the calculated region connected with the sliding anchor (along the axis y) is 8.8 cm (four calibers behind the anchor, three calibers ahead of it, plus the length of the anchor itself). The height of the calculation region (along the x axis) is 7.0 cm (half the height of the anchor, half the height of the rail, plus half the height of a dielectric above the rail). The width of the calculation region (along the z axis) is 3.5 cm (half the width of the anchor, a protruding part of the rail, plus the width of a dielectric behind the rail). The channel height (the caliber) is 1 cm, the anchor length (along the y axis) is 1.8 cm, and the rail height is 3 cm. A body with a mass of 2.5 g is accelerated with an initial velocity of 500 m/sec. The initial temperature is 290 K. The material of the anchor is Al and the material of the guides is Cu. Parameters of the calculation region and materials are the same as parameters from [3–5].

In the calculations, subregions differing in the material were split into additional difference cells: along x $(3 + 12 + 5 + 5)$, along y $(5 + 12 + 5 + 4)$, and along z $(12 + 5 + 4)$. As a result, the spatial region contains 46,332 edges of the grid, 42,671 faces, and 13,650 cells.

Figure 3 presents the specified time dependence of the total current and the calculated time dependences of concentrated characteristics of the acceleration process, namely, of the maximum temperature in the region, velocity, and coordinate of the anchor.

From Fig. 3b and c it follows that, at the time instant t = 0.21 msec and v = 1 km/sec, the maximum temperature reaches 920 K, i.e., the anchor melting begins. Subsequently, at the time instant t = 0.35 msec and the velocity v = 1.8 km/sec, the maximum temperature is already 2720 K, i.e., boiling of the anchor material begins. This is also indicated by a peculiar kink on the graph of the maximum temperature (Fig. 3b). Boiling and melting originate at the rear (in the movement direction) angular point of the anchor, which is in contact with the rail. The total current

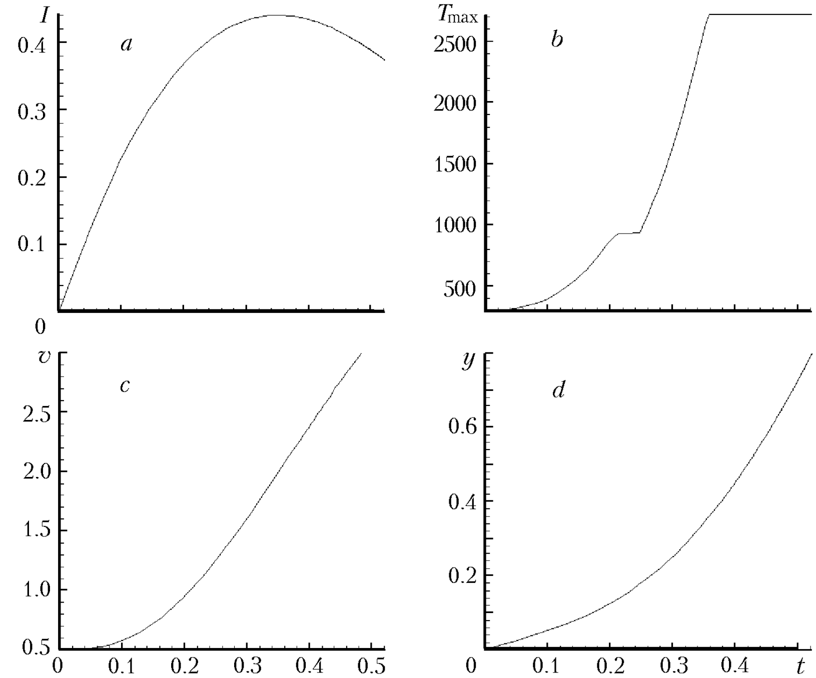

Fig. 3. Input current (a), the maximum temperature of the anchor (b); the anchor velocity (c); and the anchor coordinate (d).

TABLE 2. Dependence of Instants of the Beginning of Phase Changes on the Number of Grid Cells

Version number	Time of the beginning of melting of the anchor material, msec	Time of the onset of boiling of the anchor material, msec	Number of cells of the difference grid in the upper half-plane of a U-shaped anchor
1	0.225	0.358	$6·(6 + 4)·6 = 360$
2	0.212	0.344	$8·(9 + 5)·8 = 896$
3	0.207	0.359	$12·(12 + 5)·12 = 2448$

(Fig. 3a) varies by a specified law. The anchor flies out with a velocity of about 3 km/sec. The experimental time of existence of the dependable electrical contact of metallic type cited in [3, 4] corresponds to the time before the beginning of melting, and the onset of boiling corresponds to the time of complete breaking of the metallic contact. The exit velocity is also in agreement with the experimental value.

According to the main idea of modeling, which is the study of erosion, of greatest interest is the temperature distribution. The temperature increase in the conductor is promoted by the increased current density in it.

Comparison of the density distributions with the temperature distributions (for details, see [4, 5]) shows that heating of the rear and lateral parts of the anchor surface in the contact plane is caused by high values x and y of vector components of the current density. The current is mainly distributed over the anchor surface.

The problem is solved using a spatially uniform model without an explicit isolation of singularities. The fact that in the considered problem the current is concentrated at rear angular points of the accelerated body (where singularities of the solution [6] take place) causes heating in corresponding sections.

Let us examine the time of onset of phase changes as a function of the number of cells of the difference grid (Table 2). The calculations show that an increase of the grid fineness leads to concentration of the heat source (the Joule heating) at angular points of the accelerated body, which affects the time of the beginning of the material melting. However, the temperature dependence of the material produces such a current redistribution where by the onset of boiling does not vary in proportion to the grid spacing. The electric conductivity of the material (with account for phase changes) decreases with heating, and the currents shift to the regions with a higher electric conductivity, where

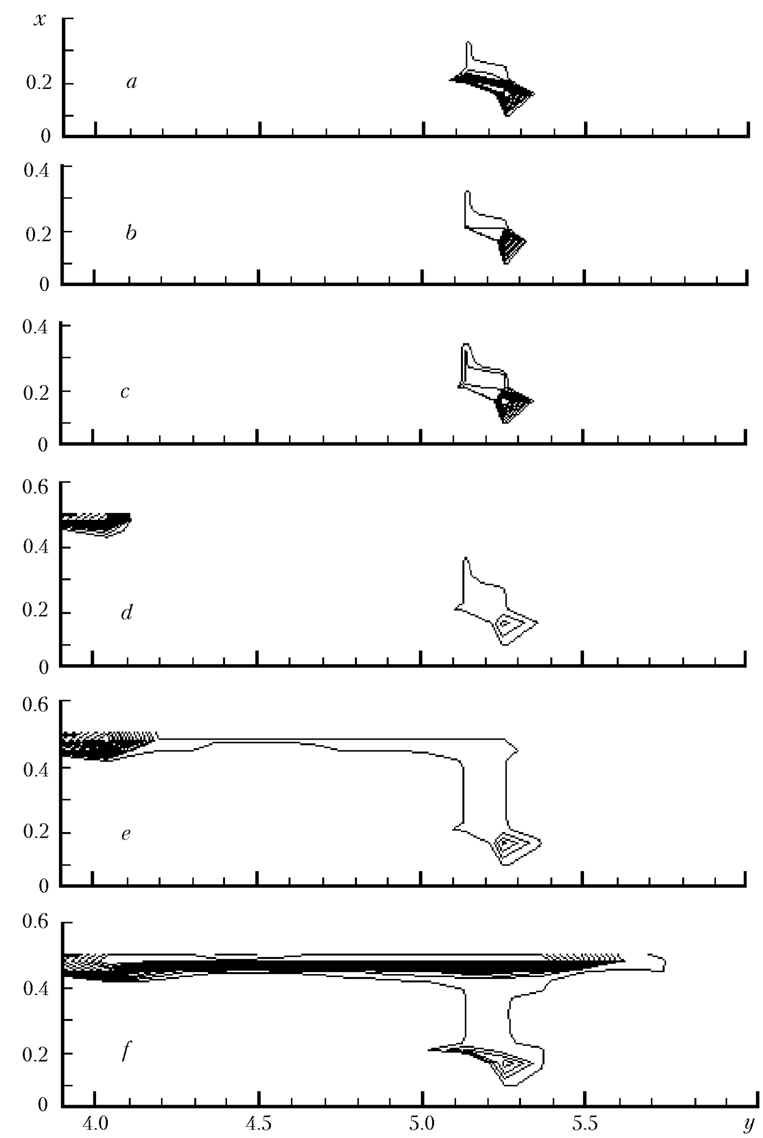

Fig. 4. Temperature distribution over the anchor (above 920 K) in sections z = const at t = 0.518 msec (the termination of acceleration): a) z = 0.33 mm (near the plane of symmetry of the accelerated body), b) 1.04, c) 1.84, d) 2.70, e) 3.67, and f) 4.72 (at the end of the region occupied by the accelerated body).

in its turn heating occurs. The employed model taking account of the current redistribution makes it possible to diminish the dependence of temporal boundaries of the material boiling on the grid spacing and to obtain good agreement with experimental data.

Investigations show that acceleration of a U-shaped body frequently entails its breaking down into several parts [3, 4]. The current passing over the anchor leads to transformation of the energy of the electromagnetic field into the thermal one and to the origination of the Joule heating, which results, among other things, in phase changes. In the considered case, the Joule heating can cause a partial or complete destruction of the accelerated body. Let us examine in more detail the possibility of through melting of the material of the accelerated U-shaped body.

Figure 4 presents isotherms above the level of the melting point (920) K) in the sections of the accelerated body by planes z = const. The pictures represent the part of the calculation region taken up by the accelerated body. Isotherms show that heating of the anchor material increases from central to lateral sections. A temperature rise can be observed in the plane of contact with the rail and in internal subregions of the anchor (at the sites of the U-shaped

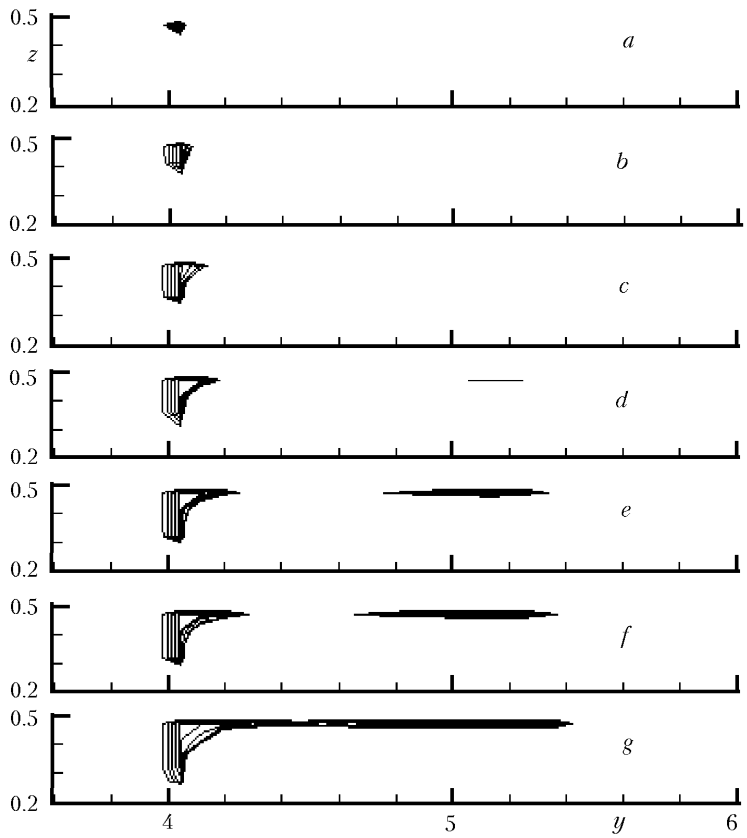

Fig. 5. Temperature distribution over the anchor (from 2400 K) in the plane of contact of the anchor and the rail along x at various time instants: a) t = 0.356 msec (the onset of boiling), b) 0.397 (boiling), c) 0.422, d) 0.448, e) 0.471, f) 0.482, and g) 0.518.

bend). All sections z = const exhibit heating at the site of bending, whose degree rises in the direction of z increase with distance from the center. Melting (just as boiling) of the material starts with regions close to the lateral (with respect to all coordinates) boundary of the accelerated body.

As seen from the presented pictures, at the end of the acceleration channel the anchor can be completely destroyed (break down into two or three parts). The current traversing the anchor is primarily concentrated on the surface of the conducting body, which is why the internal boundary (along the axis y) of the U-shaped anchor is heated to appreciable temperatures.

Since the anchor melting and boiling begin on the surface of contact with the rail, of greatest interest for studying erosion is the temperature distribution over this surface at various time instants. The maximum breaking of the metallic contact is noted in the sections where the material boiling occurs. Figure 5 shows isotherms from a temperature level of 2400 K to the maximum value in the section by the plane x = const (the plane of contact of the anchor and the rail) at various time instants. Thus Fig. 5 gives the picture of propagation, over the anchor surface, of the boiling wave which corresponds to maximum erosion of the material (erosion can also occur in melting) and to breaking of the electrical contact of metallic type.

As seen from the constructed isotherms, boiling begins at the rear (in the movement directions) angular point of the anchor and mainly propagates over the lateral surface. After some time, boiling begins also in the forepart of the anchor, subsequently moving on both sides to the center of the region.

The performed investigations are among few studies that involve measurements at the stage of transition of contact in the railtron from a metallic phase to a so-called "hybrid" contact. From the obtained picture of the three-dimensional distribution of vector and scalar fields, of greatest interest are temperature distributions in the region of contact of the anchor and the rail where evaporation of the material and degradation of the metallic contact occur. The boiling point of the anchor material is first reached at the rear of the contact surface. Further on, boiling propagates in the direction of the anchor movement as a crescent-shaped wave.

The calculated results are in qualitative and quantitative agreement with the experimental data. The experimental time of the dependable electrical contact of metallic type is in favorable agreement with the calculated time of the instant of the beginning of melting. The onset of boiling of the anchor material corresponds to the time of disturbance of the metallic conductivity. The geometric picture of through melting of the material, obtained numerically, is also close to the experimental one. The picture has been presented of through melting of the material of the accelerated body causing the anchor to split into several parts. This effect accounts for the results of destruction in the experiment. The beginning of phase changes in the calculation region has been examined as a function of the parameters of the difference grid. Various methods of modeling the material evaporation have been presented, and methods for improving the stability of the solution of the three-dimensional problem for Maxwell equations have been developed for the case of a high-temperature electrical contact.

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NOTATION

A, eigenvector of the matrix M (in dimensionless form); **B**, magnetic induction vector, T; I, current, MA; M, matrix of the system of algebraic equations obtained with the difference approximation of Maxwell equations; t, time, msec; T_{max}, maximum temperature, K; v, velocity, km/sec; x, y, and z, Cartesian coordinates, cm; λ, eigenvalue of the matrix M. Subscripts: max, maximum; min, minimum.

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